University of Saskatchewan

Department of Computational Science
CMPT 260 Final

Time: 3 hours 8 Defintions Pages April 20, 1989

Marks

1. Convert the following sentences into logic, using appropriate logical connectives, universal and existential quantifiers, and propositions and/or predicates.

3 a) If i>j, then i-1 > j, else i*j=0.

3 b) If it rains next month, all farmers will be happy, else some farmers will be in financial difficulty.

3 c) It is a nice day, if either it is clear, or it is cloudy only if it is not windy and not raining.

4 2 a) Prove $((A \wedge B) \rightarrow C) \Leftrightarrow \neg((A \wedge B) \wedge \neg C)$

6 b) Simplify the following well-formed formula

$$\neg \exists x[(\neg P(x) \wedge \forall z \neg Q(x, z)) \vee (\neg P(x) \wedge \neg R(x))].$$

10 3. Prove for all A, B, C,

$$(A \cap B) \cup C = A \cap (B \cup C) \quad \text{iff} \quad C \subseteq A$$

Note: Do not use Venn diagrams or membership tables for your proof.

4. Consider the relation R given by

aRa, aRb, bRa, bRc, cRb, and dRd.

5 a) Find $R \circ R = R^2$.

5 b) Find the transitive closure of R.

5. Let $S = \{1, 2, 3\}$, and suppose that the power set of S is denoted by P(S). Define relation e on P(S) by e(A,B) iff $|A| = |B|$.

10 a) Prove that e is an equivalence relation.

5 b) Explicitly give the equivalence classes formed by e on S.

5 c) Prove or disprove the following: for any A, B, X, and Y

$$(e(A,X) \wedge e(B,Y)) \rightarrow ((A \cap B) = (X \cap Y)).$$

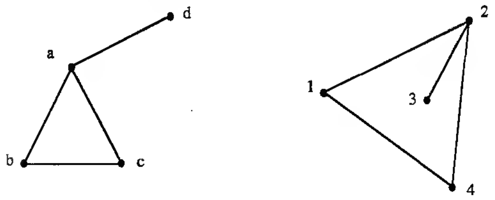
6. Consider the relation R on $\{a,b,c,d,e,f\} \times \{1,2,3,4,5,6\}$ defined by

$R = \{ (a,3), (b,1), (c,2), (d,2), (d,4), (e,6), (f,4) \}$.

3 a) Which (one) tuple needs to be deleted to make R a function?

6 b) After the tuple in part (a) is removed, is the function 1-1? Is the function onto? Justify your answer.

6 7. Consider the following two graphs



Give a 1-1 and onto function (i.e. a bijection) f from $\{a,b,c,d\}$ to $\{1,2,3,4\}$ such that x is adjacent to y (i.e. there is a line from x to y in the first graph) iff f(x) is adjacent to f(y) (i.e. there is a line from f(x) to f(y) in the second graph). Note that such a function is called a graph isomorphism, and the existence of such a function is used to prove that the two graphs are identical except for the labels on the vertices.

7 8. Suppose Sentential is an array of characters such that locations 1 through n (inclusive) contain a sentential form corresponding to some simple-precedence grammar. Suppose that the boolean functions eq, gt, and lt are defined to test whether a pair of symbols satisfies the relations $\cdot =$, $\cdot >$, and $<$ respectively. (These are given so you can just use them.) Give an algorithm (in Pascal-like notation) to find and print out the handle of the sentential form.

10 9. Consider the following grammar

A ::= CA | BcC

B ::= ab | aA

C ::= abc

where A, B, and C are non-terminals, and a, b, and c are terminals.
Find $\cdot =$ and $<$, using $(<) = (\cdot =) \circ L^{+}$. Note that the above grammar is not simple-precedence, but the two relations can still be calculated the normal way.

10. Write Prolog predicates to do each of the following. Do not use any of the builtin predicates of Prolog.

3 a) Suppose that the Prolog database contains facts of the form

person(name(smith, tom), "214 9th Ave. N, Saskatoon", "618417394").

where the first argument of name specifies the surname, and the second argument of name specifies a given name, and the third argument of person specifies the social insurance number. Give a Prolog rule for a predicate called id that succeeds for valid surname and social insurance number pairs. For example id(smith, "618417394") should succeed.

6 b) Remove the last element of a list.

10 c) Express the relation that the third argument (a list) consists of the initial segment of the first argument (a list) up to but not including the first instance of the second argument. If the second argument does not occur in the first argument, then the third argument equals the first argument. Hence

initial_seg([1,2,3,4,5], 4, [1,2,3]). succeeds

initial_seg([1,2,3,4,5], 4, [1,2]). fails

12 11. Put the following well-formed formula into clausal form. Show your work.

$$[\forall x(P(x) \rightarrow \exists yR(y))] \vee \neg [\exists x(Q(x) \vee \forall zP(x,z))] \wedge [\exists vS(v)].$$

8 12. Find the resolvent of the following two clauses. Assume that a and b are constants.

$$\{ B(g(X), X, X), C(b,X) \}$$

and

$$\{ B(Y, a, Z), D(Z, Y, g(X)) \}.$$

10 13. Show that the following set of clauses are unsatisfiable

$$\{ \{B(x)\}, \{\overline{B(x)}, C(x)\}, \{\overline{C(a)}, D(b)\}, \{\overline{C(c)}, E(d)\}, \{\overline{D(x)}, \overline{E(y)}\} \}.$$

15 14. Prove the correctness of the following program

$\{ n \geq 0 \wedge \forall j ((0 \leq j < n) \rightarrow A[j] \leq A[j+1]) \}$

```
i := 0;
while i≤n do
        begin
        B[n-i] := A[i];
        i := i + 1;
        end;
```

$\{ \forall j ((0 \leq j < n) \rightarrow B[j] \geq B[j+1] \wedge (0 \leq j \leq n) \rightarrow B[n-j] = A[j]) \}.$

Total =155 Have a good summer.